

# Statistical Methods and Reasoning for the Clinical Sciences

Evidence-Based Practice



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# Preface

Over the past 20 years or so, we have seen the rapid rise and awareness of the importance of evidence-based practice (aka EBP) in the clinical sciences. This is a movement that has focused attention on the importance of using both empirical data analysis and clinical expertise for making better and more accurate clinical decisions, such as diagnosis, evaluation, assessment, and so on. This type of research methods clearly requires more specific types of statistical techniques that go beyond and above the traditional techniques taught at school. Unfortunately, the statistical methods that we all learned are not “evidence-based”; therefore, such traditional methods have contributed to a widespread misperception. Because of the importance of the inclusion of clinical expertise to fulfill the mission of EBP, we all need to relearn how we quantitatively combine both statistical and clinical components into a single conclusion. In other words, we need to learn “How to measure the strength of evidence both statistically and clinically.” One of the most effective statistical methods to accomplish this seemingly impossible task is to use EBP statistical methods called Bayesian statistical analysis. Although Bayesian statistical methods have received an increasing amount of attention in the statistics and mathematics literatures in recent years, the use of Bayesian methods are still relatively limited in clinical decision making, especially in the fields of speech-language pathology and audiology.

Personally, I have experienced and witnessed the urgency of the use of EBP

statistical methods/Bayesian methods through a series of short courses and research seminars that I have conducted at the annual American Speech-Language Hearing Association (ASHA) conventions over the past five years. There is an ever-growing number of the clinical professionals who attended my short courses, including students, practitioners, and researchers. They have become aware of the importance of evidence-based statistical methods and interested in learning more about such methods because they have finally realized that the statistical methods in current use are not useful for clinical decision making. Unfortunately, they still have to use such traditional methods for their research, mainly because there is no alternative method that they know and they can apply. Furthermore, almost all of them still believe that the effectiveness of a treatment is always contingent on the traditional statistical benchmark of  $p$ -value of 0.05. To make the matter worse, most of them do not even know the true definition of  $p$ -value or the meaning of “measuring the strength of evidence.” If we want to move forward to EBP statistics, there is a serious need to teach them what EBP statistics is all about and how to use them. It must start somewhere, and we need to start sooner. This is my initial motivation of writing this book. Please do not get me wrong. A good working knowledge of traditional statistical methods is still important and fundamentals of EBP Statistical method. Knowing both types of methods certainly deepen one’s statistical and clinical

cal knowledge to a much larger extent in every aspect. So, this textbook explains and teaches what constitutes EBP statistical methods in philosophical, clinical, and mathematical perspectives.

Special features of this book include but are not limited to:

1. Provide feedback review, key terms and concepts, calculation guides, exercise questions, and several appendices to illustrate such topics as math review, clinical applications of statistical methods, calculation of  $p$ -values, calculation of statistical power, measuring disorder occurrence, sampling techniques, flowchart for traditional statistics versus Bayesian Statistical methods in hypothesis testing, single subject design, and writing a proposal for the ASHA convention nomogram.
2. Provide several clinically relevant case studies to deepen readers' knowledge and promote their learning to a larger extent.
3. Cover all necessary statistical topics for clinical professionals, such as descriptive methods, probability, and inferential methods, to improve their scientific literacy.
4. Give a greater emphasis on EBP statistical methods such as Bayesian statistical methods. It helps readers explore alternative methods that are extremely useful and powerful in clinical decision making.
5. Give clear conceptual distinction between EBP and non-EBP methods and limitations of traditional non-EBP methods, and explain why EBP is more clinically relevant and superior for clinical decision making?
6. Give step-by-step methods to show readers how to analyze data and interpret the result clinically.
7. Provide interpretations of statistical significance and clinical significance through several relevant and interesting examples in each chapter.



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# 5

## Part 1. Measuring Relationships: Correlation

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### General Overview

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To this point, we have been mainly concerned with a single variable or *univariate* characteristics of a sample. However, it often is the case that the purpose of a research investigation is to determine whether a statistical relationship exists between two or more variables (*bivariate* or *multivariate*) and/or whether it is possible to predict one or more variables based on other variables. For example, a clinical researcher may want to know if viewing cartoons with aggressive and violent content is related to aggressive behavior in children with a particular disorder, or whether negative political ads are related to voter apathy, or if a person's locus of control is related to his or her attitude toward academic performance. Variables related to one another in a systematic way are said to be *corelated* or *correlated*.

Often, the discovery of a relationship between variables can lead to the ability to predict one from the other. For example, if the relationship between state-trait anxiety inventory score (STAI) and Profile of Mood

State score (POMS) among autistic children is strong, we may be able to predict a child's POMS score from the child's STAI score. Because prediction is one of the major goals of the clinical and behavioral sciences, the discovery of such a relationship becomes a very important outcome of research.

The relationship between two variables is usually determined by a statistical measure known as **correlation**. The technique commonly used to predict one or more variables based on other variables is known as **regression**. In this chapter, we present the **scattergram**, also known as a *scatter diagram* or *scatterplot*. A scattergram is essentially a line graph that indicates the direction and magnitude of the relationship between two variables. In a scatterplot, the values of one of variable, in this case *drug dosage*, are listed along the vertical axis (*y*), with the lowest value at the bottom, and the values of the other variable, *reaction time*, listed along the horizontal (*x*) axis, with the lowest value placed at that the extreme left. Each pair of values is represented by a **plot point**, in this case a dot. For example, Subject C received a drug dosage of 3 and had a

reaction time score of 4.0. To plot these two scores, we locate the value 3 along the drug dosage axis ( $y$ ) and the value 4.0 on the reaction time axis ( $x$ ). At the intersection where an imaginary horizontal and a vertical line meet (here represented by dashes), we place a plot point, as shown in Figure 5–1. This one dot represents the two measures of Student C.

A *scattergram* is a line graph that indicates the direction and magnitude of the relationship between two variables by plotting pairs of scores along the vertical and horizontal axes for each individual. Each pair is represented by a dot, known as a *plot point*.

When the relationship between two variables ( $X$  and  $Y$ ) is such that an increase in one is accompanied by an increase in

the other or vice versa, we say that they have a straight line or **linear relationship**. The direction of the linearity indicates the type of relationship between the two variables. If an *increase* in a score on one variable,  $X$ , is accompanied by an *increase* in a paired score on the other variable,  $Y$ , the variables are said to have a **positive correlation**. Height and weight, fear and motivation, SAT scores and grade-point average, and stress and blood pressure are examples of variables that tend to be positively related. On the other hand, if an *increase* in the values of  $X$  is accompanied by a *decrease* in the values of  $Y$ , the variables are said to exhibit a **negative correlation**. Car speed and fuel consumption, exercise and weight, and anxiety and performance are examples of variables that usually reflect negative correlations.

When a scattergram shows that for every *increase* in the score value of one

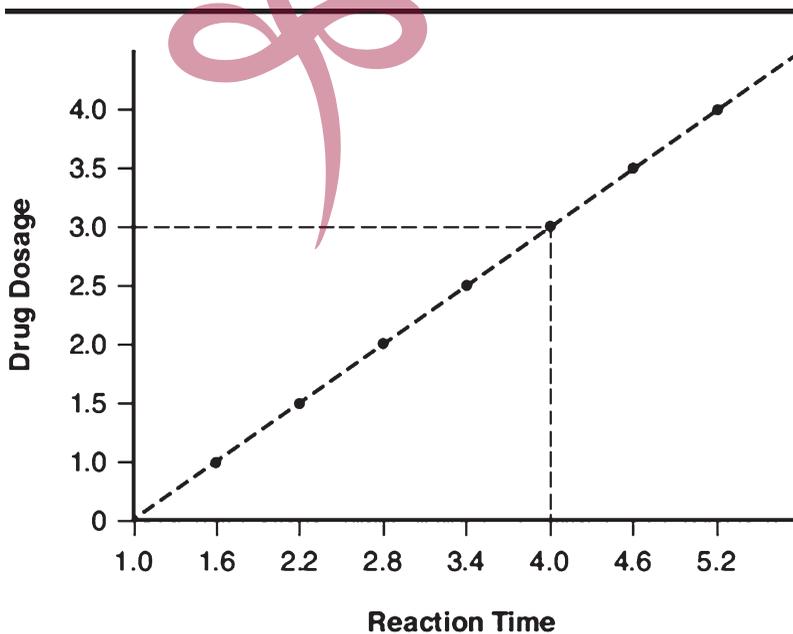
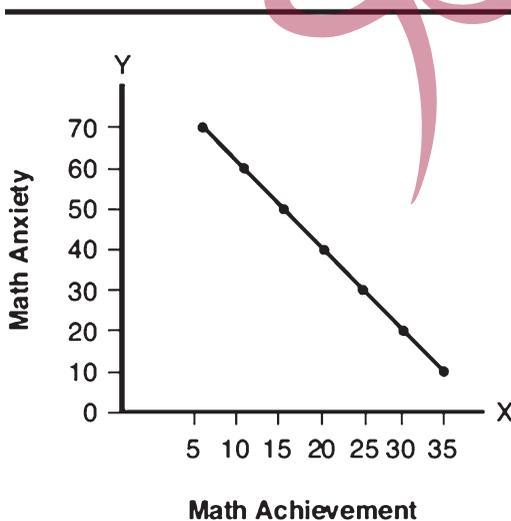


Figure 5–1. Scattergram of drug dosage and reaction time variables.

variable is accompanied by a corresponding *increase* in the score value of the other variable, the relationship between the two variables is said to have a *perfect positive correlation*. In the scattergram in Figure 5-1, the changes in drug dosage values are exactly proportional to the changes in reaction time values. Thus, we say that there is a perfect positive correlation between drug dosage and reaction time. A scattergram depicting a *perfect negative correlation*, in this case between the variables *math anxiety* and *math achievement*, is represented in Figure 5-2. Here the math anxiety scores are inversely related in a proportional way to the math achievement scores, where an *increase* in a math anxiety score is accompanied by a commensurate *decrease* in a math achievement score. In scattergrams of perfect correlations, the plot points are aligned exactly in a straight line that runs through all of the dots.



**Figure 5-2.** Scattergram of perfect negative relationship between math anxiety and math achievement.

A *linear relationship* between two variables exists when a distinct change in one is associated with a similar change in the other. When the change in the variables is in the same direction, the relationship is said to be *positive*; when the change in the variables is in opposite directions, the relationship is said to be *negative*.

### Correlation Coefficients

In reality, there are few variables that are perfectly related, mostly what we observe are variables with relationships that range somewhere between zero and 1. The plot points in scattergrams for such variables reflect trends in direction rather than exact straight lines. While useful as a visual representation of the direction of the relationship between two variables, a scattergram is limited in its ability to express the magnitude of relationships that are neither perfectly positive nor perfectly negative. To express the magnitude of the relationship between two variables statistically, we need a numerical index that represents the degree to which a correlation exists. The statistic that expresses the degree to which two sets of data are related is known as a **correlation coefficient**.

A range of correlation coefficients along a continuum starts from negative one (-1), to zero, to positive one (+1). The sign of the coefficient indicates direction of the relationship between two variables; the numeric value expresses the magnitude of the relationship. A coefficient of -1 or +1 indicates a perfect relationship while one of -.15 or +.15 suggests a weak

relationship. When there is no relationship between two variables, the coefficient is equal to zero (0).

Hypothetical scattergrams of five types of correlations are shown in Figure 5–3.

The numerical expression summarizing the size and direction of a statistical relationship is known as a *correlation coefficient*.

There are cases when two variables may be related in what is known as a **curvilinear relationship**. In such cases, both variables begin changing in the same direction but end up changing in opposite directions. Consider the two variables, fear

and motivation, mentioned above as having a positive relationship. In general, as the level of fear increases, the motivation for change also increases. However, if the level of fear continues to increase beyond a certain point, there is a tendency for the motivation to decrease or cease, as in cases where individuals suppress the root of the fear or are “paralyzed” by it. Thus, what starts out as a positive relationship between two variables ends up as a negative relationship, as depicted in the inverted U-shaped curve in Figure 5–3(d). On the other hand, consider the variables’ age and car value. At the outset, the variables are negatively related, that is to say, as a car ages its value decreases. However, at some point in time an old, deval-

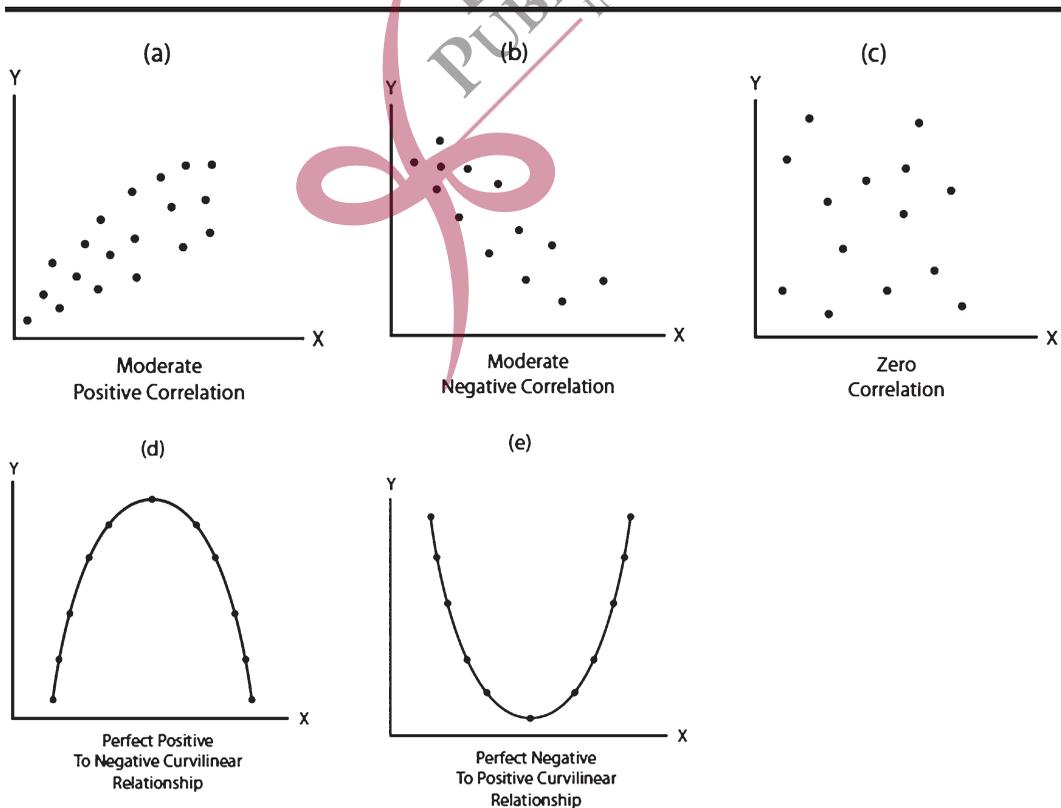


Figure 5–3. Hypothetical scattergrams depicting various types of correlations.

ued car becomes a “classic” or antique, and its value appreciates as it ages. Thus, what starts out as a negative relationship ends up as a positive one, as shown in the U-shaped curve in Figure 5–3(e). In this text, we are concerned only with methods of computing correlations for straight-line, linear relationships.

When the positive association between two variables becomes negative or vice versa, the variables are said to have a *curvilinear relationship*, characterized by a U-shape or an inverted U-shape ( $\cap$ ) curve.

Two of the more common methods for deriving correlation coefficients are the *Pearson product-moment correlation method* and the *Spearman rank-order method*.

## Pearson Product-Moment Method

The Pearson product-moment correlation coefficient, denoted by the symbol  $r$ , is probably the most frequently used coefficient to express the relationship between two variables. There are three assumptions or conditions that must be satisfied when applying the Pearson product-moment method.

1. The data must consist of paired  $X$  and  $Y$  values for each member of a sample.
2. The variables must be continuous and come from normally distributed populations.
3. The relationship between the variables must be linear.

If the conditions for assumption 2 and 3 cannot be satisfied, the Pearson  $r$  is an inappropriate measure and other techniques must be used.

### Scenario

Suppose the behavioral science researchers and clinical investigators of speech-language pathology are interested in determining if the variables *voice attraction* and *self-disclosure* are correlated. That is to say, is what we disclose about ourselves related to how vocally attracted we are to someone? We administer and score two survey instruments and obtain the following voice attraction ( $X$ ) and self-disclosure ( $Y$ ) scores for eight young adults, illustrated in Figure 5–4. While the scattergram in Figure 5–4 indicates a strong positive relationship between the two variables, we need to calculate a coefficient to objectively describe the magnitude of the correlation.

### Calculating the Pearson Coefficient $r$

The most common method of deriving  $r$  is given in Formula 5–1.

#### Formula 5–1

$$r = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\sqrt{[n(\Sigma X^2) - (\Sigma X)^2][n(\Sigma Y^2) - (\Sigma Y)^2]}}$$

We shall refer to this technique as the *raw score method*. Although seemingly complex and daunting, the computational procedure is relatively simple and similar to the raw score methods used in calculating variance and the standard deviation. To compute  $r$ , we need six values:  $n$  (sample size),  $\Sigma X$  and  $\Sigma Y$  (the sums